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CORRELATION OF TURBULENT FLOW RATE-PRESSURE DROP DATA FOR NON-NEWTONIAN SOLUTIONS AND SLURRIES IN PIPES

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Abstract—Isothermal and non-isothermal flow rate-pressure drop data in turbulent flow through smooth pipes have been obtained for non-Newtonian fluids, including aqueous solutions of polymers and aqueous suspensions of titanium dioxide. It has been found that the friction factor, f, is a function of a new form of Reynolds number, Re_B, based on the parameters A, x and w of Bowen's correlation, viz.

$$\tau_w D^x = A\bar{u}^w$$

where τ_w is the wall shear stress, $\bar{u}$ the mean velocity, D the pipe diameter; A, x and w are experimentally derived parameters which characterise the fluid.

INTRODUCTION

The available design procedures for laminar, transitional and turbulent flow of non-Newtonian fluids through smooth pipes have been comprehensively reviewed by Bowen (1961). He found that although most of the correlations which had been proposed claimed a generalised application they failed to correlate available data except in the limited range over which they were tested and the discrepancy outside this range was usually large. He concluded that the existing design procedures were only applicable to certain fluids which obeyed some of the classical constitutive equations.

Bowen studied the available data which encompassed a variety of fluids and a wide range of pipe diameters and he proposed that most of the turbulent data, irrespective of the rheological behaviour in laminar flow, could be correlated by a relationship which contained the wall shear stress, τ_w, the pipe diameter, D, and the mean velocity, $\bar{u}$, in the form:

$$\tau_w D^x = A\bar{u}^w, \qquad [1]$$

where A, x, w are constants to be determined experimentally. This method requires experimental flow rate-pressure drop data with at least two different pipe diameters in turbulent flow. It involves, first, plotting τ_w against $(8\bar{u}/D)$ logarithmically. It is well known that laminar flow data will all fall on one line, irrespective of pipe diameter, if there is no slip at the wall and no time-dependency effects. The turbulent data will branch out from the laminar line for each pipe diameter as shown in figure 1. The slopes of the laminar and turbulent data are characteristic of the fluids. For each pipe diameter the slopes of the lines in the turbulent region for a given fluid are almost identical, i.e. a set of parallel lines are obtained. This slope is w.

One of the methods for evaluating the exponent x is to calculate the average values of $\Delta P/\bar{u}^w L$ for each pipe from the turbulent data or by reading a τ_w value for each pipe at a constant $\bar{u}$ from a logarithmic plot of τ_w against $\bar{u}$ and calculating $\Delta P/\bar{u}^w L$. Then $\Delta P/\bar{u}^w L$ is plotted logarithmically against D and the slope of the straight line obtained is $-(1+x)$. The constant A may be found from [1] once x and w are known or from the intercept of the logarithmic plot of $\tau_w D^x$ against $\bar{u}$ at $\bar{u} = 1$.

Harris (1967, 1968) studied the usefulness of Bowen's method by using the data of Dodge (1957), Thomas (1960) and Shorbagi (1967) and he found it superior to any other available method to-date. Harris & Quader (1971) also discussed the usefulness of Bowen's method for

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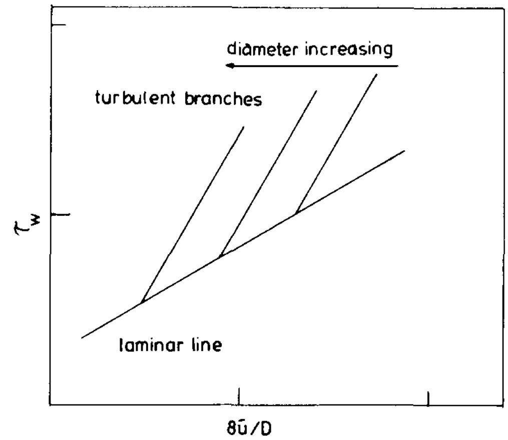

Figure 1. Typical $\tau_w - 8\bar{u}/D$ plot on log-log scale.

design purposes and Harris & Wilkinson (1971) included the Bowen parameters in a momentum and heat transfer analogy of the Taylor–Prandtl type for non-Newtonian fluids.

This paper presents further evidence of the value of Bowen's method for predicting pressure drop in isothermal turbulent flow through smooth tubes for a range of time-dependent non-Newtonian fluids, including slurries, and also extends the method to non-isothermal turbulent flow.

EXPERIMENTAL RESULTS

The pressure drop-flow rate characteristics of various non-Newtonian fluids in smooth pipes under isothermal and non-isothermal conditions were measured and the fluids were also characterised by the accepted techniques under laminar flow conditions. The details of the experimental equipment, procedures and data may be found in Quader (1972).

Pressure drop measurements in turbulent flow were carried out on two tubes of internal diameters, 1.27 and 1.91 cm. Non-isothermal pressure drop data were taken simultaneously with heat transfer measurements.

Two kinds of time-independent pseudoplastic fluids were used, namely aqueous solutions of polymers and aqueous suspensions of solids. Table 1 lists the values of the Bowen constants obtained from isothermal pressure drop-flow rate data along with the power law constants K, n or K', n', obtained from laminar flow measurements, for all the fluids studied. Cellofas is a commerical sodium carboxyl methyl cellulose made by ICI and B-10, B-300 etc. indicate different grades having different molecular weights. Tiona WD is a commerical titanium dioxide made by Laporte Industries Ltd. A preservative was added to the polymer solutions to prevent bacterial break down and a small quantity of polycarboxylic acid was added to the suspensions to improve the dispersion properties.

In order to evaluate the constants in [1] the logarithmic plots of τ_w and $(8\bar{u}/D)$ and τ_w and $\bar{u}$ for all the fluids used in this work were prepared from the isothermal pressure drop-flow rate results. Only a few of the data were taken in the laminar and transition regions. The slopes of the plots gave the exponent w in [1]. On $\tau_w - \bar{u}$ plots, the data points for both tubes were often very close to each other at a constant temperature and in some cases for different temperatures also in figure 6. Typical plots of τ_w against $(8\bar{u}/D)$ and τ_w against $\bar{u}$ for 0.40 per cent Cellofas B-3500 and 25 per cent Tiowa WD are shown in figures 2–5. The exponent x was determined from the logarithmic plot of $D\Delta P/L\bar{u}^2$ against D, where it was chosen at a constant τ_w for both tubes from $\tau_w(8\bar{u}/D)$ or $\tau_w - \bar{u}$ plots. When more than two tube diameters are involved, the plot of $D\Delta P/L\bar{u}^w$ against D would give a better value of x. For some fluids the data on $\tau_w - \bar{u}$ plots fell close together and a very small value of x was obtained for these which could be approximated to zero for all practical purposes. In figures 7 and 8, $\tau_w D^x$ and $\bar{u}$ are plotted logarithmically for 0.5 per cent Cellofas B-300, and 1 per cent Cellofas B-10.

Table 1. The values of the parameters *x*, *w*, *A* and the laminar flow properties *K*, *n* or *K'*, *n'*

Temp. °C	*n* or *n'*†	*K* or *K'*†	*x*	*w*	*A*
		Water			
8.5	1.0	0.001366	0.255	1.75	1.26
27.0	1.0	0.0008545	0.220	1.75	1.415
37.5	1.0	0.000688	0.205	1.75	1.47
59.4	1.0	0.0004713	0.185	1.75	1.54
		1 per cent Cellofas B-10			
8.5	1.0	0.0099	0.16	1.675	3.07
16.0	1.0	0.0076	0.14	1.675	3.25
27.5	1.0	0.00525	0.12	1.675	3.52
		0.5 per cent Cellofas B-300			
21.0	0.83	0.0558	0.080	1.38	5.24
40.8	0.83	0.0335	0.070	1.38	5.25
58.5	0.83	0.0218	0.095	1.38	4.54
		0.15 per cent Cellofas B-3500			
21.0	0.85	0.0241	0.0	1.48	4.74
40.8	0.85	0.0158	0.0	1.48	4.74
58.9	0.85	0.0101	0.0	1.48	4.74
		0.27 per cent Cellofas B-3500			
19.0	0.70	0.1085	0.0	1.38	5.40
43.8	0.70	0.0635	0.0	1.38	5.40
58.5	0.70	0.0410	0.0	1.38	5.40
		0.40 per cent Cellofas B-3500			
17.8	0.64	0.304	0.0	1.325	6.04
41.8	0.64	0.180	0.0	1.325	6.04
58.5	0.64	0.127	0.0	1.325	6.04
		5 per cent Titanium dioxide suspensions			
11.0	0.88	0.00295	0.0	1.79	4.03
39.5	0.88	0.00175	0.0	1.79	3.50
54.4	0.88	0.00141	0.0	1.79	3.38
		10 per cent Titanium dioxide suspensions			
10.25	0.84	0.00485	0.0	1.78	4.18
42.0	0.84	0.00230	0.0	1.78	3.60
59.8	0.84	0.00175	0.0	1.78	3.56
		15 per cent Titanium dioxide suspensions			
16.8	0.59	0.045	0.0	1.75	4.75
41.1	0.59	0.02075	0.0	1.75	4.30
55.7	0.59	0.0150	0.0	1.75	3.86
		25 per cent Titanium dioxide suspensions			
20.7	0.42	1.00	0.0	1.70	5.50
40.4	0.42	0.655	0.0	1.70	5.02
56.9	0.42	0.570	0.0	1.70	4.36

†For polymer solutions *n* and *K*; and for suspensions *n'* and *K'*.

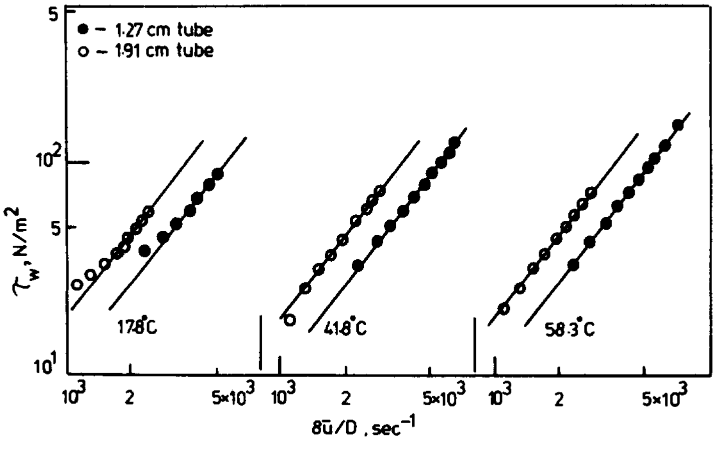

Figure 2. $\tau_w - 8\bar{u}/D$ Plot, 0.40 per cent Cellofas B-3500, $w = 1.325$.

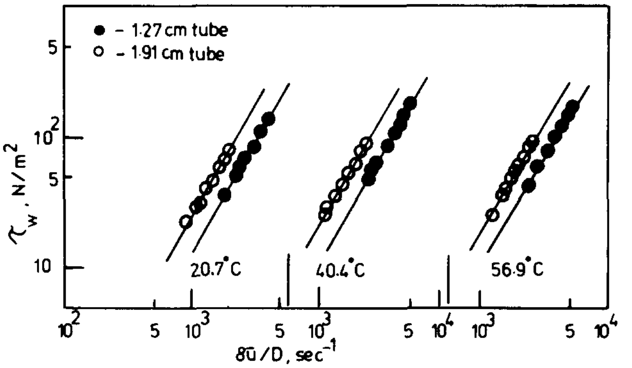

Figure 3. $\tau_w - 8\bar{u}/D$ Plot, 25 per cent Tiona WD, $w = 1.70$.

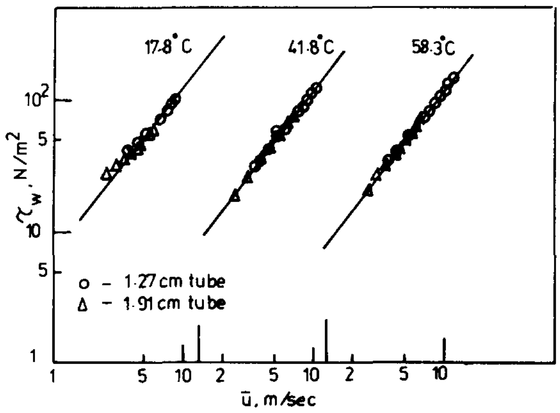

Figure 4. $\tau_w - \bar{u}$ Plot, 0.40 per cent Cellofas B-3500, $w = 1.325$.

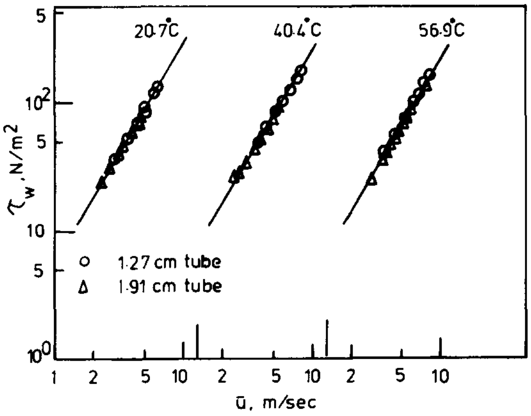

Figure 5. $\tau_w - \bar{u}$ Plot, 25 per cent Tiona WD, $w = 1.70$.

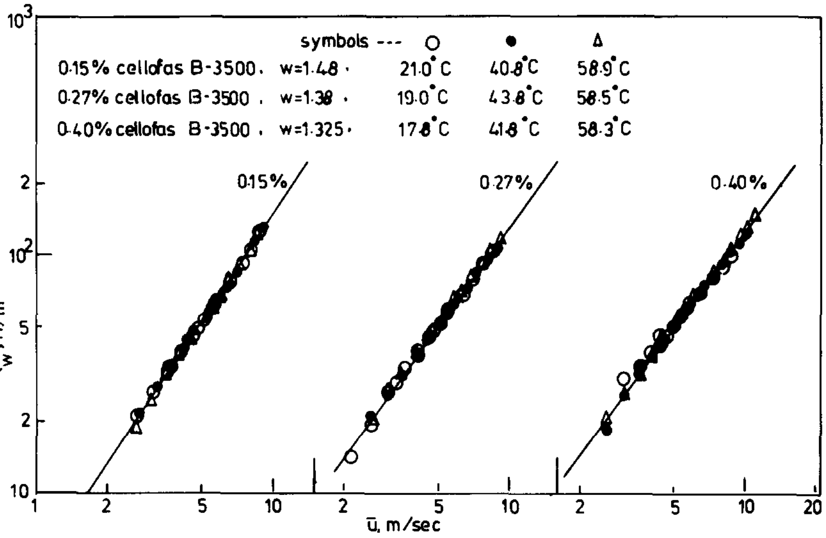

Figure 6. $\tau_w - \bar{u}$ Plot.

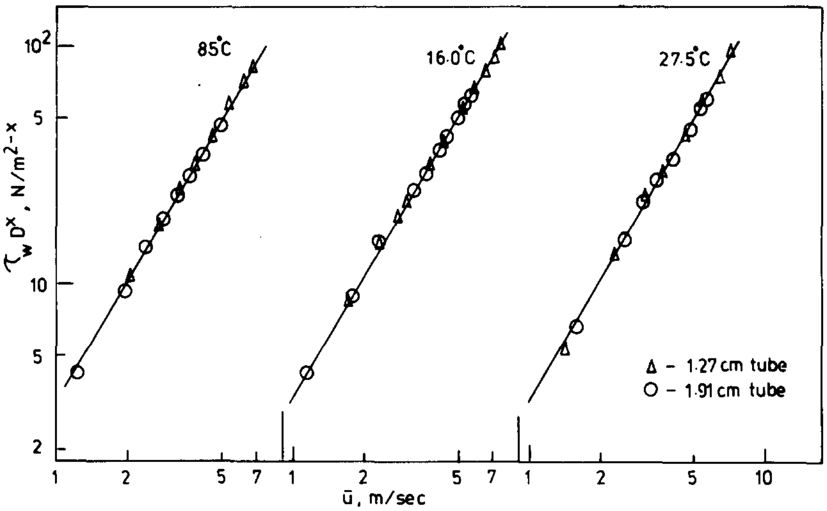

Figure 7. $\tau_w D^x - \bar{u}$ Plot, 1 per cent Cellofas B-10, $w = 1.675$.

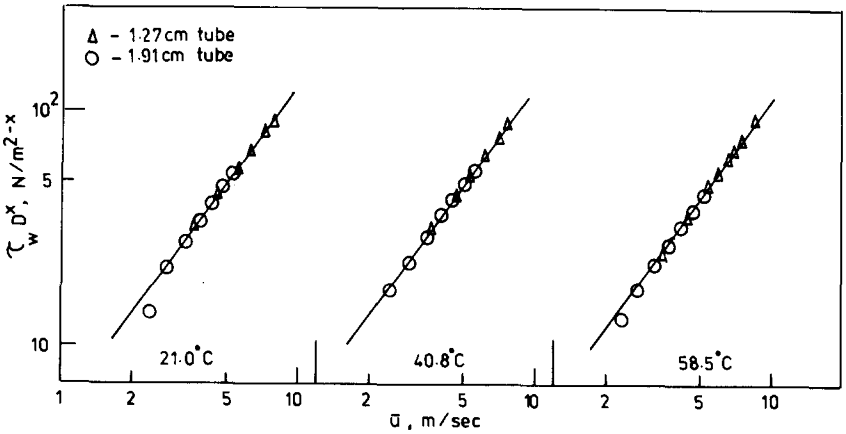

Figure 8. $\tau_w D^x - \bar{u}$ Plot, 0.5 per cent Cellofas B-300, $w = 1.38$.

CORRELATION OF PRESSURE DROP

In this method the wall shear stress is related to mean velocity and tube diameter by [1]. If τ_w is considered to be some function of $\bar{u}$, D, ρ, x, w and A for fully developed turbulent flow through smooth pipes, that is:

$$\tau_w = \phi(\bar{u}, D, \rho, x, w, A) \qquad [2]$$

then by dimensional analysis it can be shown that the friction factor, f, may be written as,

$$f = \phi\left(\frac{\bar{u}^{2-w}D^x\rho}{A}, x, w\right). \qquad [3]$$

On the other hand, by substituting the value of τ_w from [1] into the usual definition of the friction factor, i.e.

$$f = \tau_w/\tfrac{1}{2}\rho\bar{u}^2 \qquad [4]$$

we would get,

$$f = 2\bigg/\left(\frac{\bar{u}^{2-w}D^x\rho}{A}\right) = 2/\mathrm{Re}_B. \qquad [5]$$

The group $(\bar{u}^{2-w}D^x\rho/A)$ is a new form of Reynolds number based on Bowen's constants and denoted as Re_B. From [3] it is clear that f is a function of x and w on a $f - \mathrm{Re}_B$ diagram, whereas [5] suggests that $f - \mathrm{Re}_B$ diagram will be independent of x and w in the turbulent flow.

In order to find out the relationship between f and Re_B, the logarithmic plots of $f - \mathrm{Re}_B$ were drawn for all the fluids (figures 9 and 10). The data fall on a single straight line of slope -1 and the scattering of data points is very small.

Figure 11 compares the calculated values of Re_B with the experimental values for all the non-isothermal pressure drop data taken simultaneously with heat transfer for both types of fluids. $\mathrm{Re}_{B\mathrm{cal}}$ was obtained by using the values of x, w, A determined from isothermal pressure drop

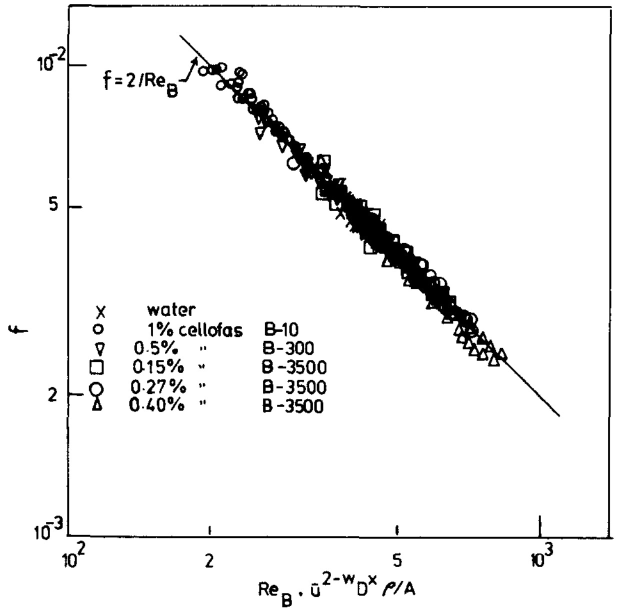

Figure 9. $f - \mathrm{Re}_B$ Plot for isothermal pressure drop results.

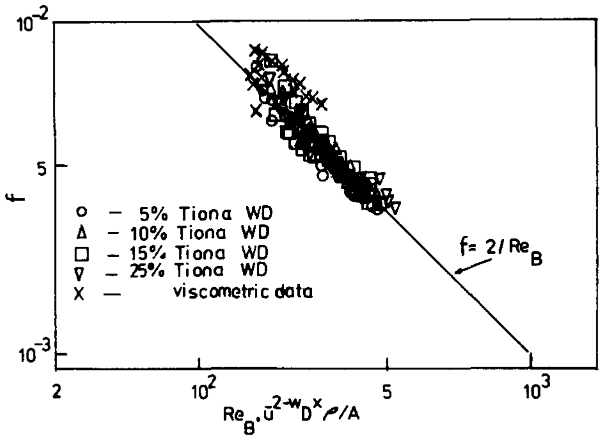

Figure 10. $f - \mathrm{Re}_B$, Plot for isothermal pressure drop results.

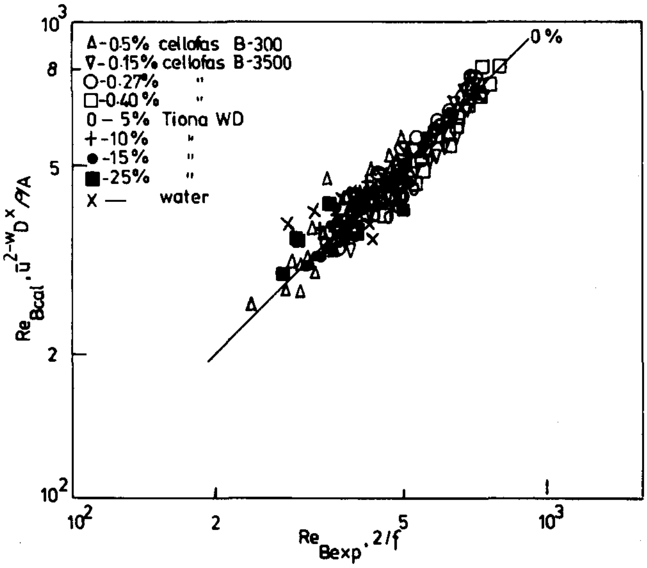

Figure 11. $\mathrm{Re}_{\mathrm{Bcal}} - \mathrm{Re}_{\mathrm{Bexp}}$ Plot for non-isothermal pressure drop results.

data and $\mathrm{Re}_{\mathrm{Bexp}}$ was calculated from [5] using the experimentally measured friction factor obtained from non-isothermal pressure drop measurements, i.e.

$$\mathrm{Re}_{\mathrm{Bexp}} = 2/f_{\mathrm{exp}}$$

where $f_{\mathrm{exp}} = \tau_w/\frac{1}{2}\rho\bar{u}^2$ in which τ_w and $\bar{u}$ are both experimental values. $\mathrm{Re}_{\mathrm{Bcal}}$ is evaluated at the average bulk temperature of the fluids.

DISCUSSION

It has been found that the exponent w of Bowen's correlation, [1], does not depend on temperature or tube diameter for a given fluid. However, the exponent x does vary with temperature but for most of the fluids studied x is very small, as seen from table 1. The values of w lie between 1.79 and 1.33 and those for x in between 0.25 and 0. For a given material w decreases with concentration. The constant A for a given fluid is rather insensitive to temperature which is surprising since A may be considered to be related to K' or K.

The plots of $\tau_w D^x$ and $\bar{u}$, figures 7 and 8 show the accuracy with which the isothermal pressure drop data are correlated by [1]. $\tau_w D^x$ and $\bar{u}$ plots were made for fluids with $x > 0.020$.

For other fluids the $\tau_w D^x - \bar{u}$ plot becomes coincident with the $\tau_w - \bar{u}$ plots, because D^x is unity when x is nearly equal to zero for the tubes sizes used in this work. The $\tau_w - \bar{u}$ plots for these fluids for which x has been taken equal to zero show that this approximation holds good as seen from figure 6. The small values of x indicate that the velocity profile in the turbulent core is flat and the velocity profile may be calculated (Harris 1967). Usually x has been found to decrease with increase of temperature. This indicates that the velocity profiles in the turbulent core get flatter as temperature increases.

When x is less than 0.04 the error in assuming it to be zero for the fluids studied in this work can easily be estimated. Over a fifty-fold change of diameter the error in the calculation of τ_w would not be more than 20 per cent.

Table 1, which lists the values of the turbulent flow parameters x, w, A along with laminar flow parameters K, n or K', n' for all fluids, shows that these parameters do not have any relationship with each other. Although the fluids have the same power law index n or n', that is, the same degree of non-Newtonian behaviour, the Bowen constants are different. It is clear that the latter are dependent on the nature of turbulence which is highly individualistic for each fluid and is not necessarily influenced by the laminar flow behaviour characteristics. The parameters x, w and A do not give any quantitative information regarding the nature of turbulence in the pipe. Nevertheless, they are useful for correlating pressure drop data and the method appears to be more reliable than any other method.

Comparison of [1] with the relationship of Metzner–Reed (1955), viz.

$$\tau_w = K'\left(\frac{8\bar{u}}{D}\right)^{n'} \qquad [6]$$

shows that this is a special form of [1] in the laminar flow condition. When $x = w = n'$ and $A = 8^{n'}K'$ [1] reduces to [6].

It should be noted that in attempting to find out the constants of Bowen's correlation only isothermal pressure drop data for a fluid should be used and care should be taken that the fluid has not suffered shear degradation during the measurements. Such degradation may easily be checked if flow curves for samples collected during the experiment are determined or some of the measurements are repeated during the experiment for reproducibility.

The plots of friction factor, f, against Re_B, the Reynolds group from a dimensional analysis involving the Bowen constants, show that the data for all fluids lie on a straight line of slope -1 (figures 9 and 10). No recognisable effects of x and w are observed on $f - \mathrm{Re}_B$ plots for the fluids listed in table 1.

It has been observed that even for the same value of power law index n or flow behaviour index n', the value of Re$'$ for the end of laminar and transitional flow is not the same. However, by constructing a logarithmic plot of τ_w and $(8\bar{u}/D)$ by using both the constants of Bowen's formula and those for the laminar flow model, i.e. [6], then the limit of laminar flow can be determined at the intersection of the laminar and turbulent curves. The locating of the end of laminar regime by this method should facilitate design for non-Newtonian fluids.

The use of the parameters of Bowen's formula to calculate non-isothermal pressure drop has also been shown to give reasonable accuracy as can be seen from figure 11.

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